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# Improving the Reliability and Usability of Structural Shaping Optimization – The Contour Natural Shape Function

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## Abstract

A new approach for parametric modeling of finite element model for shaping optimization is developed. This approach only needs the finite element mesh, yet can still rebuild a reverse parametric CAD model for efficient modeling. Special care was paid to make sure the modeling parameters were reduced, but still had enough flexibility for shaping definition. With similar algorithms of the Natural Shape Function, this approach is able to keep the original mesh pattern through out the design iterations, therefore eliminates the response noise produced by re-meshing. Unlike other methodologies, the current one only needs to define the parameters on the contour of the model. Cutting the whole domain into several subdomains is not necessary. Therefore it is called the Contour Natural Shape Function.

**Keyword** Nonlinear Programming, Numerical Design Optimization, Structural Shaping Optimization, Parametric modeling, Contour Natural Shape Function

## 1. Introduction

Structural shape optimization methodologies are fairly mature. For example, the problem has been approached using stochastic methods like the genetic algorithms (Rajan, 1995, Chen and Rajan, 2000, and Chen, 2001). However, one of the most important issues in structural shape optimization is still how to mathematically model shape changes using design variables. From history, most of the methods can be categorized into four different general approaches.

The first approach uses the nodal coordinates as the design variables directly (Zienkiewicz and Campbell, 1973, Ramakrishnan and Francavilla, 1975). For this approach the number of design variables is very large, and the quality of the finite element mesh may deteriorate. Some researchers also try to add connections and constraints between nodes (Huang et al., 1981, Holzleitner and Mahmoud, 1999), in order to keep better mesh quality as well as reduce the total number of design variables. This approach can be built upon existing FE packages without CAD modeling tools available, but intensive user input and judgement is necessary.

The second approach, the geometry-based approach, uses the controlling parameters in CAD or geometric model as design variables, and relates the nodal coordinates with the blending interpolation or parameter of the geometry (Chang and Choi, 1992, Tortorelli, 1993, Chen and Tortorelli, 1997, Hardee et al., 1999, Annicchiarico and Cerrolaza, 1999). Some researchers also consider the locations of the boundary nodes only for design variables (Seong and Choi, 1987, Botkin, 1992, Song and Baldwin, 1999). For better mesh quality, there are also algorithms available to re-distribute the internal nodes. Hwang and Fleury (1993), for example, re-located the nodes based on optimizing the shape quality of each element. Another approach is to use the Laplacian formulation for re-distributing the internal nodes (Kodiyalam, Virendra and Finnigan, 1992, Lindby and Santos, 1994, Banichuk et al., 1995). Only small changes on the boundary are allowed. The approach mentioned above will require internal information from the CAD system, therefore intensive programming effort may be necessary. Also a proper parametric model on the

CAD level is crucial. This approach sometimes suffers from the problem of inconsistent mesh pattern after re-meshing, unless a mapping mesh procedure or re-distribution of mesh nodes is implemented. Sometimes controlling the error estimate of the FE analysis has been used to minimize the inconsistency caused by re-meshing (Banichuk et al., 1995).

Two other methods seem to be popular in the commercial structural optimization packages, like MSC/NASTRAN (MacNeal-Schwendler Corporation, 1996), and GENESIS (Vapderplaats Research & Development, Inc., 2000). The first method, which is called the fictitious load method or the Natural Shape Function approach (Belegundu and Rajan, 1988), uses an auxiliary structure, and applies fictitious load as a design variable. The structural shape and mesh are then updated according to the deformation of the auxiliary structure. This approach has relatively good flexibility in the variation of structural shape, but the selection of the fictitious load and boundary conditions depends on the user's experience. An improved version of this approach, called the Hybrid Natural Approach (Rajan, Chin and Gani, 1996), replaces fictitious load with displacements, and couples with geometry parameters to simplify the process of deciding design variables. Also the concept of velocity field matrix (Choi and Yao, 1986) has been used to avoid real-time calculation of the deformed auxiliary structure. The improved version of this approach will require parametric information from the CAD model. Similar modeling methods can also be found in the field of computational fluid dynamic optimization, which constructs analog truss elements on the edges of the elements (Nielsn and Anderson, 1999).

The remaining approach uses domain elements or super elements to define the perturbed mesh domain with a few controlling key nodes (Kodiyalam, Vanderplaats and Miura, 1991). The perturbed basis was calculated beforehand and the selection of the design variables was simplified. However, the flexibility of the structural shape variations will depend on the pre-defined mathematical model of the domain elements and sometimes the mesh still deteriorates near the intersecting boundary of elements.

Most of these methodologies mentioned have a few things in common. First, attempts have been made over the years to reduce the input effort needed for controlling and deciding the final shape of the structure. These attempts were achieved by allowing the users to pick a few nodes only for altering the coordinates, and interpolating the coordinates of the rest of the nodes in some special formulation. The nodes that are assigned as the design variables directly are called the *Controlling Nodes* in this paper. All other nodes, which are affected by the *Controlling Nodes*, are called *Passive Nodes*.


## 2.    The Contour Natural Shape Function

## 2.1    Difficulty of the Geometry-Based Approach

The goal of this research was to develop a methodology that would take advantage of the existing methods. The tasks were then to minimize the user input effort but still retain reasonable flexibility in the design variables. The algorithm was also expected to use finite element meshes only without CAD information. The reason for minimizing the user input data and retaining reasonable flexibility is obvious. The reason for limiting the available information to finite element mesh only will be explained below. Note that these comments are suitable for the author's working environment but may not be universally true for all situations.

As mentioned before, the geometry-based approach requires a robust parametric CAD model which can be very difficult to make, if not impossible. There are several reasons for this.
1.    Frequently in the industry a surface-based modeler will be used. This increases the difficulty of making a parametric geometry model.
2.    The CAD model is usually made to include all details. For a meaningful mesh to be created, these models need to be de-featured, otherwise the computational expense for analysis will be unaffordable. It is questionable whether the parametric CAD model can still exist after the de-featuring.
3.    Some CAD software today allows the meshing to be done within the CAD software. However rarely can all the necessary boundary conditions and loads be completely applied in the CAD module. More frequently the geometry will be exported into the FEA package, modified, and meshed. Then the boundary conditions and loads will be applied and the analysis performed. Or the user may create the mesh with the CAD software and export it into the FEA package. Then the loads and boundary conditions are applied and the problem solved. Either way means interruption in the process.

4.	A parametric CAD model requires the modeling parameters be defined when building the model. However these controlling parameters and geometry feature may no longer be suitable after entering the detailed design.

It is obvious that, unless the CAD package can have the same analysis strength as the FEA package, or the geometric modeler inside the FEA package is as powerful as the sophisticated CAD, hoping for a robust and easy integration between these two modules is not realistic at this time.

There are other reasons not to favor the geometry approach too. Figure 1 shows the traditional concept of a closed-looped optimization, where the parametric CAD model is closed inside the optimization loop. Figure 2, however, shows a slightly different version of an optimization loop, where there is an iterative loop inside the loop of the parametric CAD model. The advantage of the latter methodology is that right before and after the modification of the CAD model, engineering judgement is allowed. Also, if the scope of the optimization loop is reduced, effort on maintenance of the code is greatly reduced, and so is the risk of unexpected termination. Coding the mesh perturbation algorithms inside the FE package also makes it easier to access the analysis response database. This approach takes advantage of a good parametric CAD model and also reduces the restriction of the CAD modeling as well.

For the reasons stated above, the geometry-based approach was not considered during this study.

## 2.2	Objectives of the New Algorithm

Table 1 shows the comparison of the techniques indicated earlier between non-CAD based algorithms for structural shaping optimization. A few interesting facts can be observed. First, while the user usually expects less input required for the optimization task, it is obviously conflicting with the demand for flexibility on design shapes. However, since in practical design there is usually limitation on the changeable feature, eventually the flexibility of design shapes is pre-defined. Therefore a predefined flexibility of design shapes is actually acceptable.

Second, if a pre-defined flexibility is acceptable, then the Hybrid Natural Approach [23] seems to be the best choice. However, getting detailed information from a sophisticated CAD system is usually difficult, unless the CAD system itself supports this feature. Also, analysts usually need several iterations to build a satisfactory mesh which compromises both accuracy and computer power. When the analysis is sophisticated, the association between the parametric CAD model and the mesh is likely to be destroyed, since it will become more and more frustrating for analysts to compromise between meshing and modeling capability of the package. From the author's experience, there is usually only finite element mesh existing after some iteration. The original CAD model, even if carefully built, does not play a significant role anymore.

The objective here is to overcome this obstacle but still take advantage of the Hybrid Natural Approach. This is called the Contour Natural Shape Function approach. The desired features are compared in Table 1. The objectives of the current research can be summarized as follows, and the details will be explained in the following sections.
1.	Reserve the flexibility of the candidate shape in Natural Shape Function approach.
2.	Reduce the number of user-input nodes.
3.	Reduce the number of design variables compared with the original Natural Shape Function approach, therefore also reduce the dimension of the design space.
4.	Build a parametric model without CAD data or information. Thus the design variables should be set up in a CAD-like fashion. It should be easily built on any existing finite element packages without accessing or interference with the internal database and code.
5.	Allow different parametric model settings upon one preliminary design without excessive effort.

| | Node-Based | Natural Shape Function | Domain Elements | Hybrid Natural Approach | This Paper |
|---|---|---|---|---|---|
| Controlling Nodes | All Nodes | **Boundary** | **Boundary** | **Boundary** | **Boundary** |
| Passive Nodes | **NONE** | NONE | All Nodes | **NONE** | **NONE** |
| Domain Division | **NONE** | **NONE** | YES | **NONE** | **NONE** |
| Design Variables Basis | NONE | NONE | **YES** | **YES** | **YES** |
| Continuity Across the Whole Structure | NO | **YES** | NO | **YES** | **YES** |
| Design Shape Flexibility | **Infinite** | **Infinite** | Pre-defined | Pre-defined | Pre-defined |
| CAD Information | **NO** | **NO** | **NO** | YES | **NO** |

Table 1 Comparison of Modeling Methods of Interests

### 2.3 The Mesh Perturbation Algorithms

The basic concept of the Hybrid Natural Approach for controlling the design shape, is first to divide the boundaries of the model into several connected segments. Each segment is then controlled by one CAD geometry entity and a few key points, and the perturbations of all other nodes are calculated by proper interpolation. The Natural Shape Function approach is then invoked to calculate the perturbation on the whole model.

Now assuming the CAD geometry information does not exist, it is then natural to consider rebuilding some geometry information based on the defined boundary nodes. For example, in a 2D problem, the user may pick a segment of nodes on the boundary, and then hope to define variation of these nodes based on only, say, 3, controlling nodes, for a parametric quadratic perturbation. If all nodes perfectly lie on this line, proper parameters can be assigned to each node easily, and many formulations can be used for calculating perturbation on the passive nodes based on given perturbation on the controlling nodes.

However, calculation of the parameters for each node on the curve will encounter difficulty, if the nodes do not lie perfectly on the reconstructed quadratic line. This situation is illustrated on Figure 3.

The problem is then how to assign a parameter for each passive node assuming that the line (or surface) does not perfectly match the contour. For a 2D finite element model, assuming the parametric line is to be interpolated, for example, by the Lagrange Interpolation as

$$x = \sum_{j=1}^{N} N_j(\xi) \cdot x_j \tag{1}$$

$$y = \sum_{j=1}^{N} N_j(\xi) \cdot y_j \tag{2}$$

with $(x_j, y_j)$ the coordinate of the $j$th controlling node, and $N_j(\xi)$ the corresponding shape function of parameter $\xi$. The problem of assigning the parameter $\xi$ to a node with the coordinate $(x_i, y_i)$ is then formulated as

Find $\quad \xi$

To Minimize $\quad \sqrt{(x_i - x)^2 + (y_i - y)^2} \tag{3}$

That is, to find a point on the fitted line, that has the shortest distance to an arbitrary node $(x_i, y_i)$ on the boundary. The parametric value of this node is then assigned to the corresponding passive node. This is a non-constrained minimization problem and can be easily solved by many algorithms. Actually, if the passive nodes are to be interpolated (instead of extrapolated) by the controlling nodes, it is reasonable to bound the parameter between 0 and 1. The problem then becomes a simple-bounded minimization problem.

Multiple solutions may exist if the passive nodes are too far away from the fitted curve. However, it is reasonable to ask the user to use a proper and rational fitted curve for the passive nodes. If the optimization process fails to find a reasonable solution (which can be judged by the perturbation explained later), it should always be suggested to the user to adjust the setting, either by changing the degree of the polynomial or reassigning the controlling nodes and passive nodes.

## 2.4 Calculating the Perturbation on the Passive Nodes

Once the curve or surface parameters of the passive nodes are decided, the perturbation on each *Passive Node*, based on the perturbation on the *Controlling Nodes*, can be obtained from modifying (1) and (2) as

$$\Delta x = \sum_{j=1}^{N} N_j(\xi) \cdot \Delta x_j \qquad (4)$$

$$\Delta y = \sum_{j=1}^{N} N_j(\xi) \cdot \Delta y_j \qquad (5)$$

where $\Delta x$ and $\Delta y$ are the coordinate perturbations on the calculated *Passive Nodes*, and $\Delta x_i$ and $\Delta y_i$ are the coordinate perturbations of the *Controlling Nodes* on the curve that the *Passive Node* lies on.

With the perturbations defined on the controlling nodes and the ones on the passive nodes available, the Hybrid Natural Approach can be readily used. That is, when the new contour of the structural shape is decided, the final modified structural shape can be calculate by a static structural analysis with specified displacement on all nodes of the contour. The basic process is explained below

Step 1 The user defines the sets of *Controlling Nodes*, and their corresponding *Passive Nodes* from segment to segment (or surface to surface), until all necessary controlling boundary nets are defined.

Step 2 Loop through each passive node to calculate its parameters relative to the proper domain, controlled by the proper controlling nodes.

Step 3 Loop through each active degree of freedom (DOF) of each controlling node, assume a unit perturbation and then calculate the interpolated perturbation on each *Passive Node* associated with it. If a passive node is not affected or controlled by this unit perturbation, its perturbation is set to zero. Use these perturbation values as the specified displacement on the structure, and solve the static case. The displacement on each node is then saved to one Design Variables Basis $\mathbf{X}_i$.

Step 4 The user specifies a desired perturbation on each DOF of each controlling node. These values are then used to scale its associated Design Variable Basis. This decides the influence from perturbation of one DOF of one controlling node. The final structure shape is then the linear combination of all these perturbations, which can be expressed as equation (6).

$$\mathbf{B} = \mathbf{B}_0 + \sum_{i=1}^{NDV} b_i \cdot \mathbf{X}_i , \qquad (6)$$

where $\mathbf{X}_i$ are the Design Variables Basis created in step 3, and $b_i$ are the values of the perturbations on each Controlling Node. $\mathbf{B}_0$ is the coordinate of the original structure and $\mathbf{B}$ is the coordinates of the final structural shape. Currently, the Design Variables Basis is not changed during the optimization process.

Note that, in a structural optimization program, step 1 through step 3 can be isolated outside the optimization loop. The user can repeat these steps until he or she feels the quality of the Design Variables Basis is satisfactory. The user then feeds all the information into the optimization program and step 4 is repeated inside the optimization loop. The values of the perturbation on each controlling node, $b_i$, can then be the design variables of the shaping optimization process. This approach also provides more flexibility and accepts engineering judgment before the time-consuming loop is started.

## 2.5    Design Sensitivity Analysis

The analytical design sensitivity of the current method is essentially the same as the one explained in the Hybrid Natural Approach [23]. The details will not be explained here and the interested readers can refer to the previous publication. Currently only the numerical design sensitivity is used.

## 2.6    Advantages of the Contour Natural Shape Function Approach

The advantages of this approach are
1.    The flexibility of the candidate shape in Natural Shape Function approach is preserved.
2.    The number of user-input nodes is greatly reduced.
3.    The number of design variables can be greatly reduced compared with the original Natural Shape Function approach, therefore the dimension of the design space is also reduced.
4.    This method allows easy combination with both the domain element approach and the natural shape function approach.
5.    No CAD data is necessary. Thus the design variables are set up in a CAD-like fashion. It can also be easily built on any existing finite element package without accessing or interference with the internal database and code.
6.    It is easier to define different parametric models upon the existing one preliminary design without much effort.

These characteristics of the approach are summarized in the first right column of Table 1.

## 3.    Numerical Examples

## 3.1    Square Plate with a Hole in the Center

This example is commonly used to test shaping optimization algorithms. It is a square plate subjected to uniform and equal distributed load on all edges. Figure 4 shows the one quarter symmetric finite element model and the dimension with loads and boundary conditions.

The thickness of the plate is 0.7747 mm. The distributed load $P$=10 lb/in., $E$= $2.07 \times 10^{11}$ Pa, $\upsilon$ =0.3, density $\rho = 20.3 \, \text{kg/m}^3$. Limit on the Von Mises stress is $6.89 \times 10^6$ Pa. The objective of this problem is to minimize the weight of the plate without violating the stress limitation. The changeable contour is defined as straight lines AB and FG, and quadratic lines BCD and DEF. There are totally 8 design variables. They are the x and y coordinates of C, D and E, the $y$ coordinate of B and the $x$ coordinate of F. (The number of design variables can be further reduced to four if symmetric linking is implemented).

Figure 5 shows the Design Variables Basis calculated on nodes B, C and D, in both $X$ and $Y$ directions (for other nodes, the results are similar). As can be seen in this figure, the mesh and boundaries are smoothly moved, even with just perturbation in one node and one direction.

Figure 6 shows the final optimal design. The problem took 14 design cycles to converge. Numerical gradients were used and a total of 119 function evaluations were taken (which includes 31 function evaluations and 11 numerical gradient evaluations). The final maximum Von Mises stress was 1000 psi. The final shape of the hole is a circle, which is the same as the analytical optimal solution. The weight was reduced by about 22%, and the radius of the circle is 3.82 inches, which is almost exactly the analytical optimum. This example in the previous publication [22] uses 12 design variables. It took 20

design cycles, 64 function evaluations, and 19 analytical gradients in the previous publication to achieve the same result.

### 3.2 Turbine Disk Shape Optimization

Figure 7 shows the finite element mesh of the preliminary design of a turbine disk in a gas turbine engine. The model is a two-dimensional axial-symmetric cross-section, with *X* representing the rotational axis. The disk is subjected to centrifugal load. The initial design did not satisfy the stress constraints and reduction of weight was required. The objective is to minimize the weight by changing the shape and to reduce the stress under allowable limits. Figure 6 also shows the controlling key nodes 1 through 14. The x and y coordinates of nodes 2 to 5 and 8 to 14 are defined as the design variables.

Figure 8 shows the final design after the optimization process. The weight is reduced by 22 percent of the original weight, and the maximum stress was reduced to 5 percent below the allowable value. The detailed values are not supplied here due to proprietary issue considerations.

### 4. Conclusion

An algorithm for mathematically modeling and optimizing structural shape changes was developed. Special attention was paid to reduce the user input effort but still maintain reasonable flexibility. The examples show that the proposed algorithm is able to achieve good results with less design variables in some cases, yet reduce the size of the design space. The application of the natural shape function approach provides less element distortion during the design process as demonstrated in numerical example 1. The modular development of the methodology also provides flexibility to combine with an existing finite element analysis package even when the access to the internal database is limited. Using the proposed approach, a CAD-like parametric model for design optimization is possible even though a CAD parametric model is not available. Therefore this approach can be applied into the existing industrial design process without re-inventing the whole analysis and design system.

Although only two-dimensional cases are evaluated in this paper, the application transformation to three-dimensional is trivial, since the formulation and algorithms here can be easily expanded for such situations.

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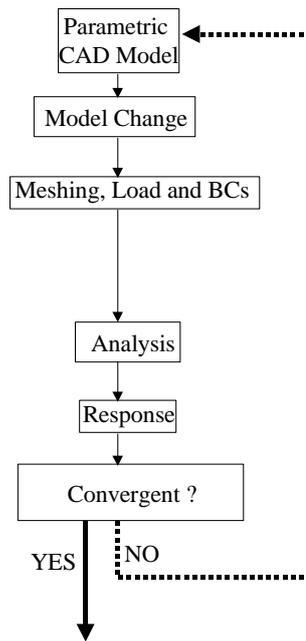
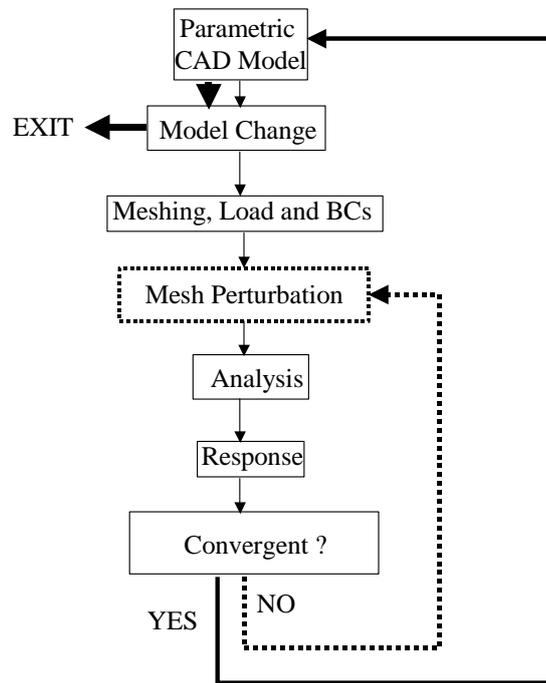Figure 1 Traditional Design Loop with Parametric CAD Model.

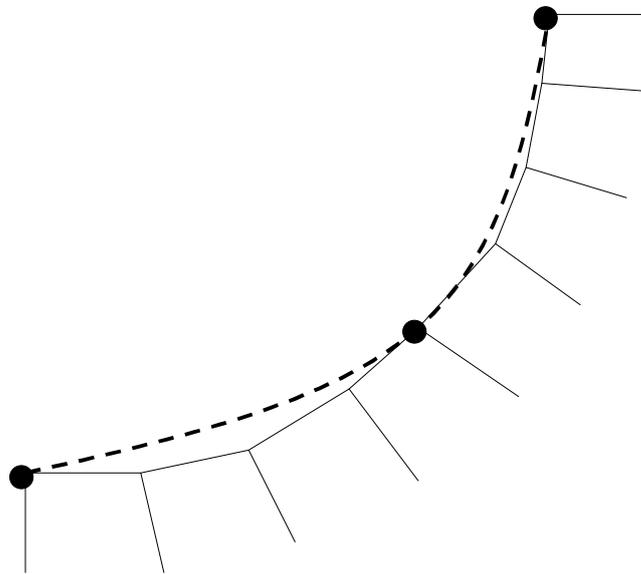Figure 2 Design Loop with Parametric CAD Model and Mesh Perturbation



Figure 3 The Re-Built Geometry Curve (the Dash Line) and the Actually Mesh Contour (solid line), The Picture Shows the Inconsistency Between the Two Boundary/Curve.
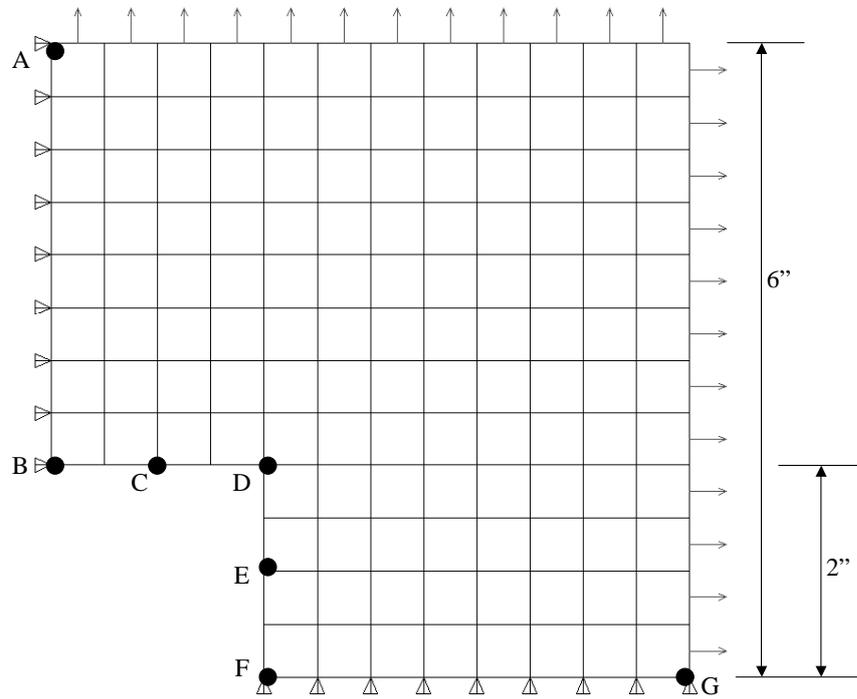
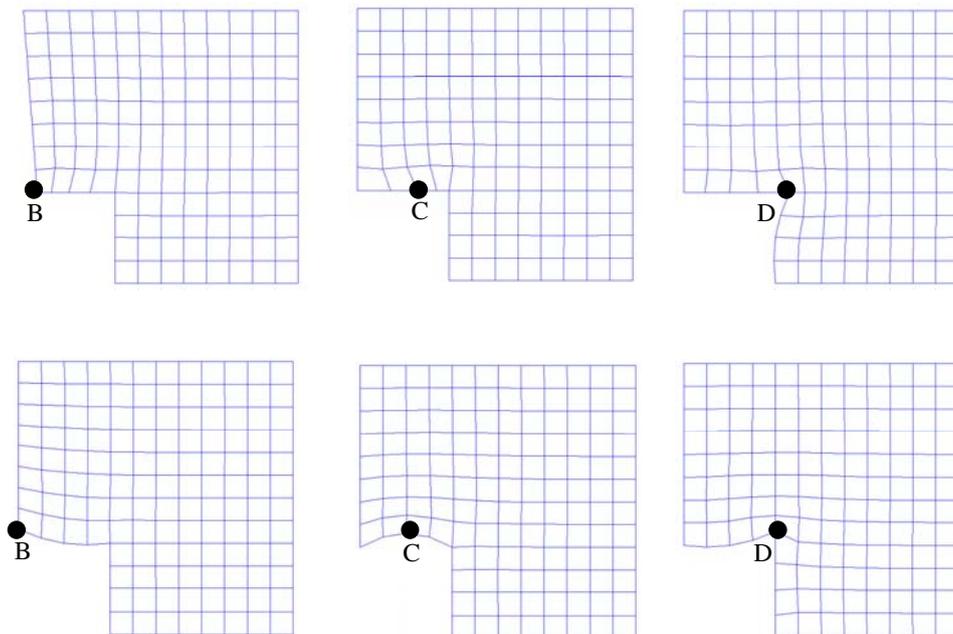Figure 4  Geometry, Finite Element Mesh, Loading and Boundary Conditions for Example 2



Figure 5 Perturbed Mesh and Design Variables Basis on Nodes B (Left), C (Middle) and D (Right), in the X direction (Top) and Y Direction (Bottom)
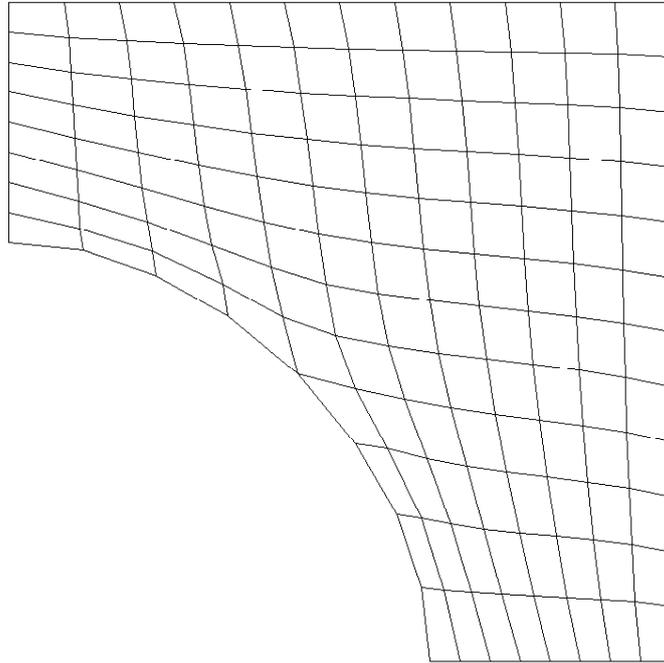
Figure 6 Final Optimum Design of Example 2 Shows the Hole Approaching to a Circle, Which is The Same as the Analytical Solution
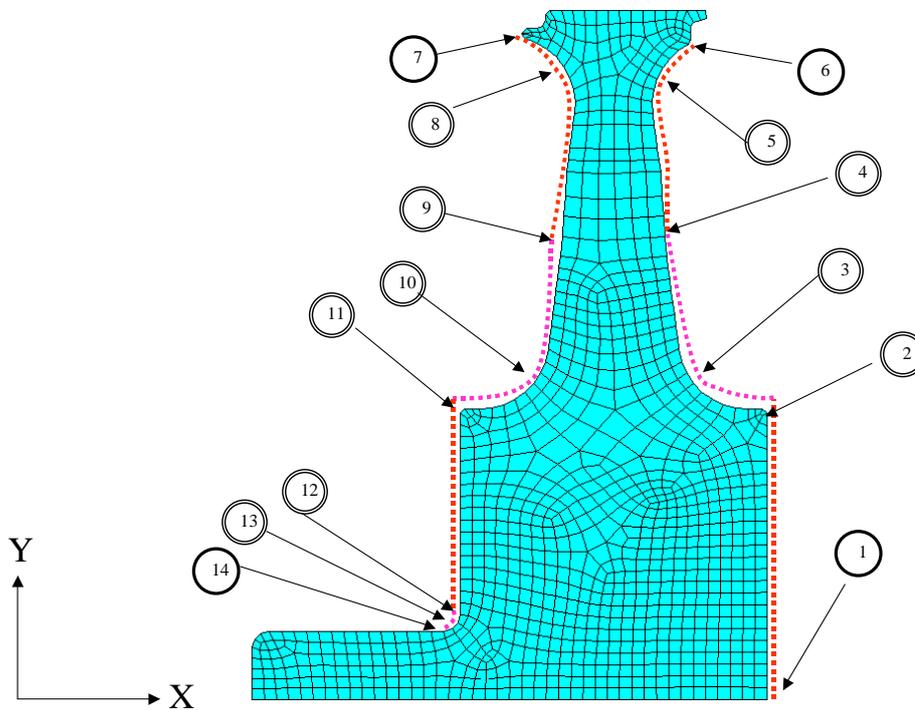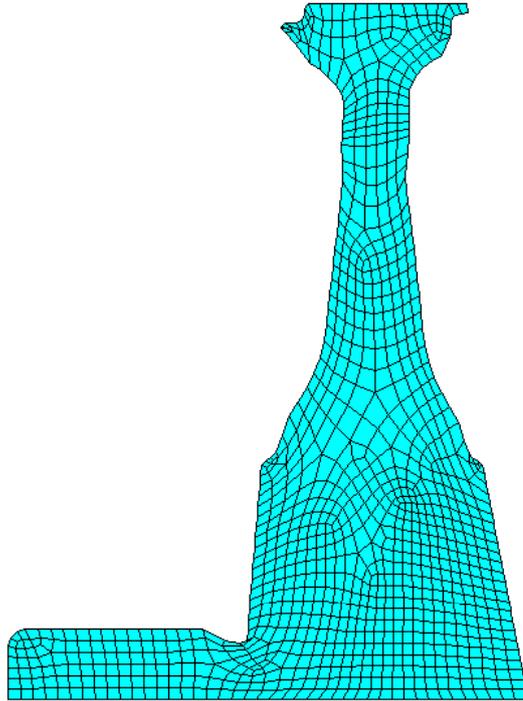


Figure 7 Initial Design and Controlling Nodes of the Turbine Disk

Figure 8 Final Optimum Design of the Turbine Disk